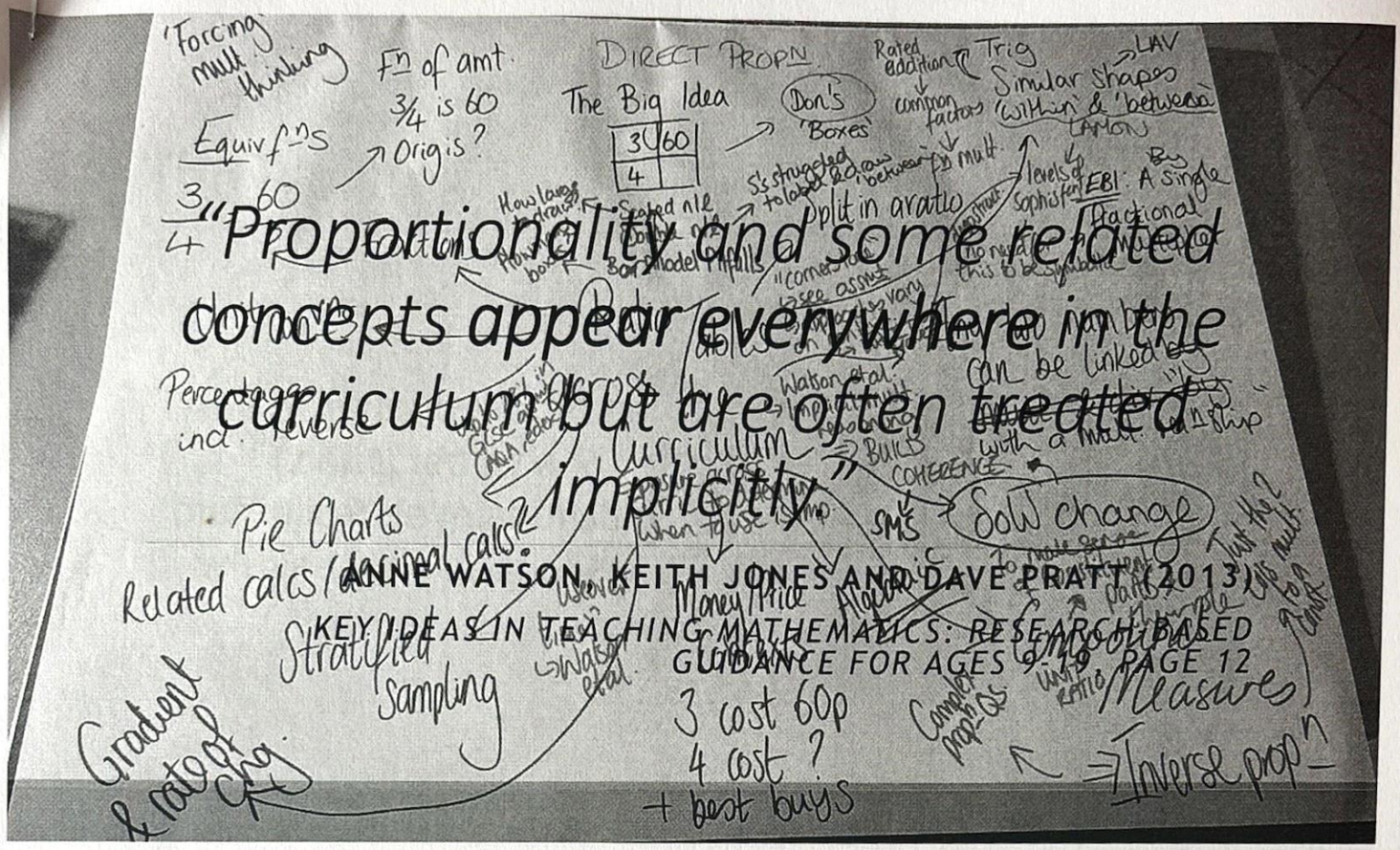
Breaking Up with Formulae: Ratio Tables are "The One"

KATHRYN CLARK
REGIONAL LEAD PRACTITIONER FOR MATHS (SOUTH YORKSHIRE),
ASTREA ACADEMIES TRUST
ARITHMATICKS@GMAIL.COM
@ARITHMATICKS *@ARITHMATICKS.BSKY.SOCIAL*

The Plan

- A recap of ratio table notation (just in case!)
- Algebraic direct proportion
- Inverse proportion intro
- Algebraic inverse proportion



"Proportionality and some related concepts appear everywhere in the curriculum but are often treated implicitly"

ANNE WATSON, KEITH JONES AND DAVE PRATT (2013) KEY IDEAS IN TEACHING MATHEMATICS: RESEARCH-BASED GUIDANCE FOR AGES 9-19, PAGE 12

Ratio Table Recap

11	7
88	56

2	8
10	40

50	20
10	4

6	8
36	48

21	14
3	2

20	8
15	6

10	6
25	15

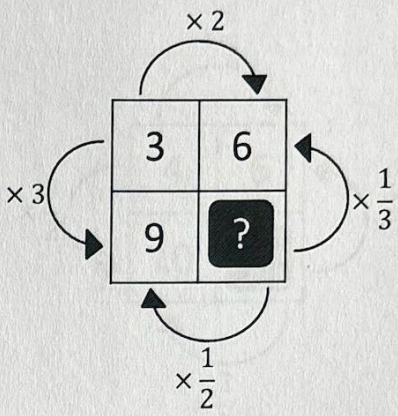
8	6
20	15

20	16
25	20

6	15
10	25

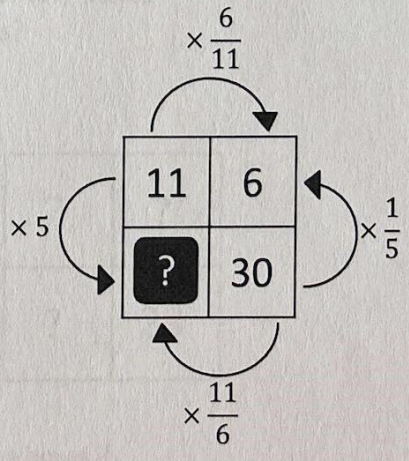
The Big Idea

Any two numbers can be connected by a multiplicative relationship with a single multiplier.



The Big Idea

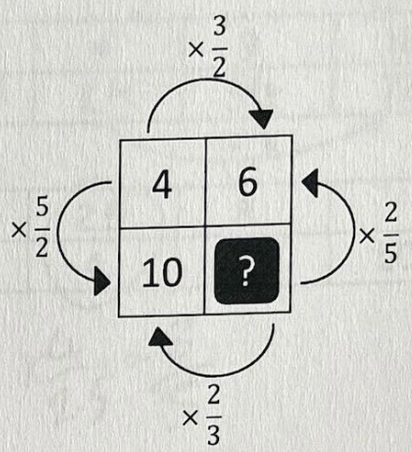
Sometimes one of these relationships is 'more difficult' than the other…



The Big Idea

Sometimes neither direction is very pretty...

But if you know about reciprocals as multiplicative inverse, you're all set!



Deeper Thinking

What **integers** can be placed in the empty boxes to create **valid ratio tables**?
How do you know?

a

15	?
?	12

b

7	?
?	23

How many ways can you complete the boxes if you can use fractions and decimals?

"Algebraic" Ratio Tables

Question 5: y is directly proportional to x.
Complete the table.

$y = 6x$

x	4	9	12
y	24	54	72

Question 6: y is directly proportional to x
Complete the table.

x	2.5	8	
y	4		50

$y = \frac{8}{5}x$

$\times \frac{4}{2.5} = \frac{8}{5}$

https://corbettmaths.com/wp-content/uploads/2019/03/Proportion-Direct-and-Inverse.pdf

Algebraic Problems

y is directly proportional to x
When $y = 12$, $x = 3$

a) Find the equation linking x and y

b) If the value of $y = 60$ what is the value of x?

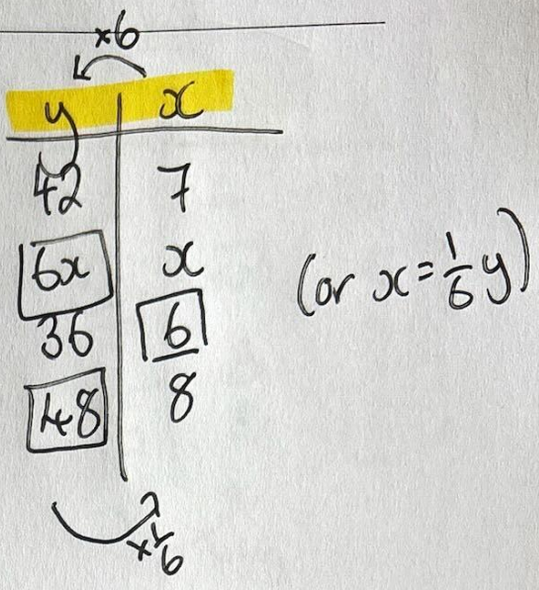
c) If $x = 5$ what is the value of y?

y	x
12	3
20	5
60	15
4x	x
y	$\frac{1}{4}y$

Your Turn

y is directly proportional to x
When $y = 42,\ x = 7$

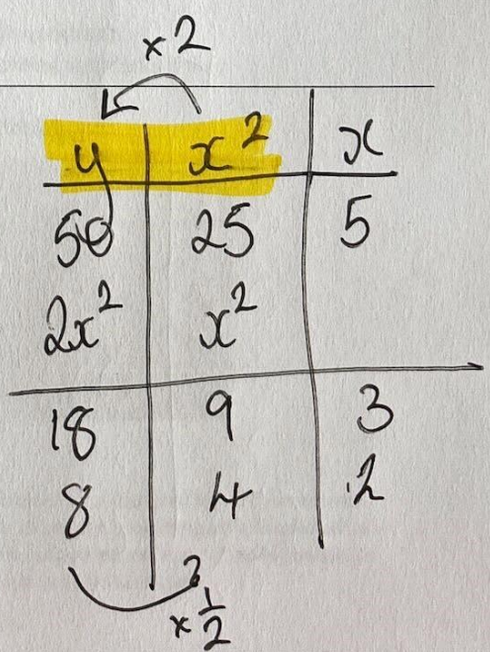
a) Find the equation linking y to x $y=6x$
b) If $y = 36$ what is the value of x
c) If $x = 8$ what is the value of y



Adding Complexity...

y is directly proportional to **the square of** x
When $y = 50\ x = 5$

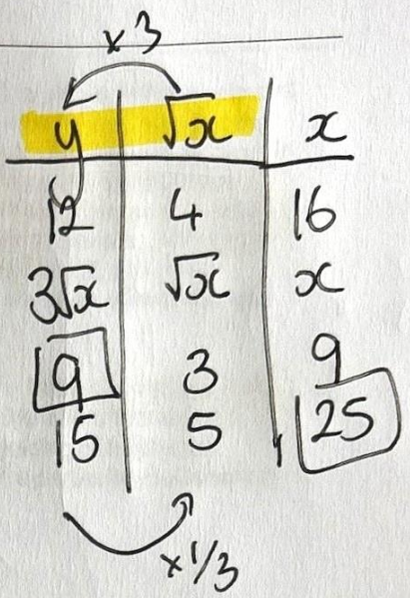
a) Find the equation linking y to x $y=2x^2$
b) Find y when x is 3 18
c) Find x when y is 8 2



Your Turn

y is directly proportional to **the square root of** x
When $y = 12$ $x = 16$

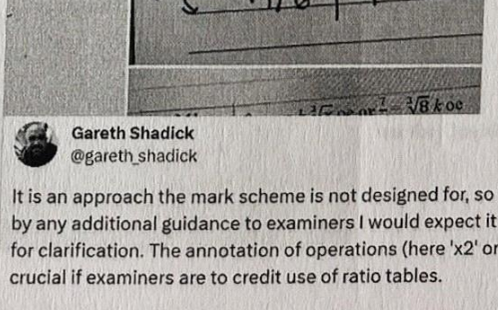
a) Find the equation linking y to x $y = 3\sqrt{x}$
b) Find y when x is 9
c) Find x when y is 15



Exams...

Amie Meek
@alcmaths

I'm a huge fan of ratio tables for algebra proportion questions.
However, I want to check if students would get working out marks if they didn't answer the q completely correctly.
Could they pick up the first M1 mark from using a table?



Gareth Shadick
@gareth_shadick

It is an approach the mark scheme is not designed for, so if not covered by any additional guidance to examiners I would expect it to referred up for clarification. The annotation of operations (here 'x2' on both sides) is crucial if examiners are to credit use of ratio tables.

1:45 PM · Sep 29, 2023 · **155** Views

JustMaths @Just_Maths · Oct 4
I'd refer this up the food chain as the first line doesn't include k
1 · 154

JustMaths @Just_Maths · Oct 4
So for a mock I wouldn't give it.
1 · 1 · 180

Amie Meek @alcmaths · Oct 5
If I'd have added an arrow with 'x k' from the cube root of x to y, would that have tempted you to give M1?
1 · 145

JustMaths
@Just_Maths

Yes

8:00 AM · Oct 5, 2023 · **43** Views

Exams...

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

[Edexcel 1MA1/1H, June 2023, Q13]

13 y is directly proportional to x.

$y = 24$ when $x = 1.5$

Work out the value of y when $x = 5$

$y =$ .....

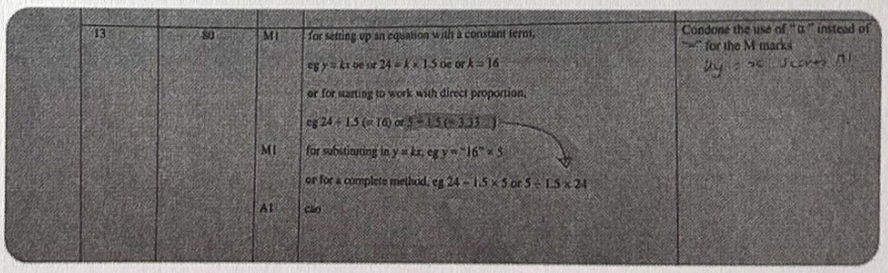
(Total for Question 13 is 3 marks)

Exams...

Rose Procter
@Rose_Rover

Follow ...

I haven't got the question paper to hand but this is Q13 from this summer's Edexcel 1H if that helps?



13	80	M1	for setting up an equation with a constant term, eg $y = kx$ oe or $24 = k \times 1.5$ oe or $k = 16$ or for starting to work with direct proportion, eg $24 \div 1.5$ (= 16) or $5 \div 1.5$ (= 3.33...)	Condone the use of "α" instead of "=" for the M marks eg $y \propto x$ scores M1
		M1	for substituting in $y = kx$, eg $y = "16" \times 5$ or for a complete method, eg $24 \div 1.5 \times 5$ or $5 \div 1.5 \times 24$	
		A1	cao	

5:00 PM · Oct 5, 2023 · **117** Views

Exams...

13 y is directly proportional to x.

$y = 24$ when $x = 1.5$

Work out the value of y when $x = 5$

[Edexcel 1MA1/1H, June 2023, Q13]

13	80	M1	for setting up an equation with a constant term, eg $y = kx$ oe or $24 = k \times 1.5$ oe or $k = 16$ **or** for starting to work with direct proportion, eg $24 \div 1.5$ (= 16) or $5 \div 1.5$ (= 3.33...)	Condone the use of "a" instead of "=" for the M marks
		M1	for substituting in $y = kx$, eg $y =$ "16" $\times 5$ **or** for a complete method, eg $24 \div 1.5 \times 5$ or $5 \div 1.5 \times 24$	
		A1	cao	

More than two variables...

a is directly proportional to b.

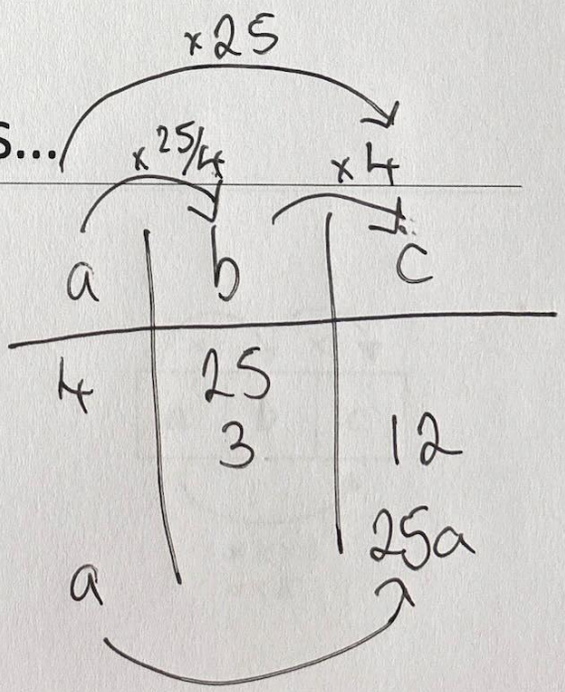
b is directly proportional to c.

If $a = 4$, when $b = 25$

and $b = 3$, when $c = 12$;

Find a formula connecting a and c

$c = 25a$



Your Turn...

x is directly proportional to y.

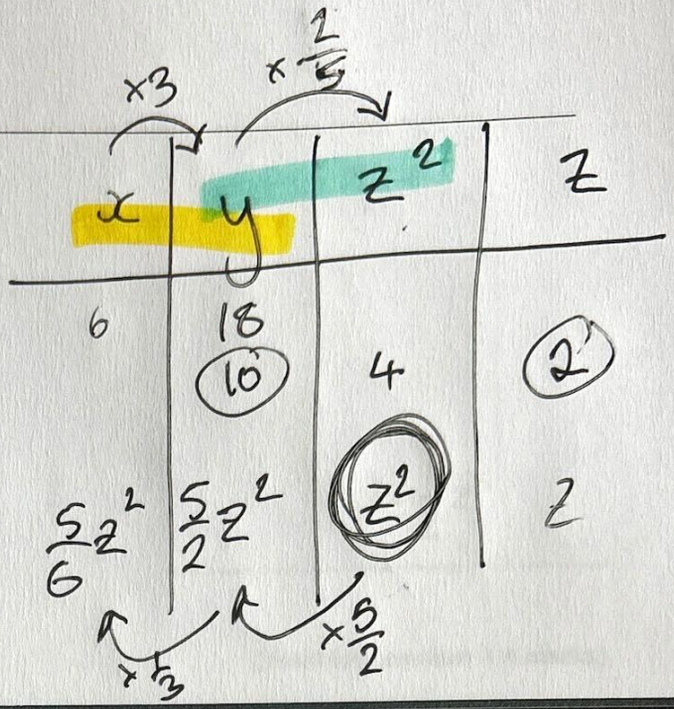
y is directly proportional to z^2.

If $x = 6$, when $y = 18$

and y = 10, when z = 2;

Find a formula connecting x and z

$x = \frac{5}{6}z^2$



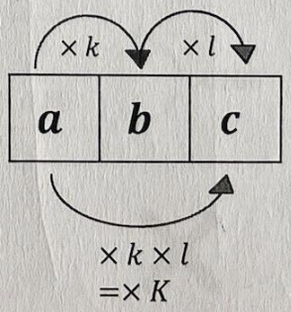
More than two variables...

In general, if there are two multipliers, k & l, such that

$b = ka$ and $c = lb$

Then we can combine these two multipliers to give a single multiplier between a and c.

$c = l(ka) = Ka$



More complexity...

[Edexcel 1MA1/3H, Nov 2021, Q17]

x is directly proportional to the square of y.
y is directly proportional to the cube of z.

$z = 2$ when $x = 32$

Find a formula for x in terms of z.

$$x = \frac{1}{2}z^6$$

.....

(Total for question = 4 marks)

Inverse Proportion

[Edexcel 1MA1/1H, Nov 2022, Q13]

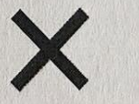
p is inversely proportional to t

Complete the table of values.

$\times \frac{1}{100}$

t	100	25	500	2
p	1	0.25	5	0.02

$\times 100$



(Total for question = 3 marks)

What is the same?
What is different?

1 The table shows the distance you can travel in various times at 30 miles per hour.

Time (min)	10	15	20	30	40	45	60
Distance (miles)	5	7.5	10	15	20	22.5	30

Constant multiplier

$\times \frac{1}{2}$

2 The table shows how long it would take to travel 30 miles at various speeds.

Speed (mph)	10	15	20	30	40	45	60
Time (h)	3	2	1.5	1	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{2}$

$\times 2$ $\times \frac{1}{2}$

Multiply to give a constant

df

$ST = 30$ $S = \frac{30}{T}$ $T = \frac{30}{S}$

Inverse Proportion

2 The table shows how long it would take to travel 30 miles at various speeds.

Speed (mph)	10	15	20	30	40	45	60
Time (h)	3	2	1.5	1	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{2}$

$Speed \times Time = 30$

$S \times T = D$

$T \times S = D$

$S = \frac{D}{T}$

$T = \frac{D}{S}$

Direct or Inverse?

1

x	3	9	12	15	30
y	60	20	15	12	6

Inv $xy=180$

2

a	4	5	8	10	15
b	7.5	6	3.75	3	2

Inv $ab=30$

3

j	3	9	12	15	30
k	60	120	150	180	330

Neither!
$k=10j+30$

4

p	4	5	8	10	15
q	3	3.75	6	7.5	11.25

$\times\frac{3}{4}$

Direct $q=\frac{3}{4}p$.

Inverse Proportion

[Edexcel 1MA1/1H, Nov 2022, Q13]

p is inversely proportional to t

Complete the table of values.

t	100	25	20	2
p	1	4	5	50

$\times 4$ $\times 4$

100

VS

$p \propto \frac{1}{t}$

$p = \frac{k}{t}$

$1 = \frac{k}{100}$

$100 = k$.

(Total for question = 3 marks)

Algebraic Problems

y is inversely proportional to x.

When $x = 4,\ y = 18$.

a) Find the value of y when $x = 6$

b) Find the value of x when $y = 6$

c) Find a formula connecting x and y

$xy = 72$ $\quad y = \frac{72}{x}$

$xy = 72$

x	y
4	18
6	12
12	6

Your turn...

y is inversely proportional to x.
When $x = 6, y = 10$

a) Find the equation connecting y and x
b) Find x when $y = 3$

$xy = 60$

$xy = 60$

y	x
10	6
3	20

Algebraic Problems

y is inversely proportional to x.

When $x = 4,\ y = 18$.

a) Find a formula for x in terms of y

Rearrange! $xy = 72$

$x = \frac{72}{y}$

b) Find a formula for y in terms of x

$\frac{xy}{x} = \frac{72}{x} \Rightarrow y = \frac{72}{x}$.

$xy = 72$

y	x
18	4

Your Turn

r is inversely proportional to t.

When $r = 6,\ t = 35$.

a) Find the value of t when $r = 15$

b) Find the value of r when $t = 0.7$

c) Find 3 formulae connecting r and t

$rt = 210$

$t = \frac{210}{r}$ $r = \frac{210}{t}$.

$rt = 210$

r	t
6	35
15	14
300	0.7

Adding Complexity

a is inversely proportional to b^2.

If $a = 3$, when $b = 10$;

Find a formula connecting a and b

$ab^2 = 300$

a	b^2	b
3	100	10

Your Turn

y is inversely proportional to $\sqrt{x}$.

When $x = 25$ $y = 9$

Find an equation connecting y and x.

$y\sqrt{x} = 45$

y	$\sqrt{x}$	x
9	5	25

Your Turn

[Edexcel 1MA1/1H, June 2017, Q13]

The table shows a set of values for x and y.

$x^2y=9$

x	1	2	3	4
x^2	1	4	9	16
y	9	$2\frac{1}{4}$	1	$\frac{9}{16}$

$\frac{9}{16}$ 16

y is inversely proportional to the square of x.

(a) Find an equation for y in terms of x.

$y=\frac{9}{x^2}$

(2)

(b) Find the positive value of x when $y = 16$

$x=\sqrt{\frac{9}{16}}$

3/4

(2)

More than two variables...

60

[Edexcel 1MA1/1H, June 2017, Q13]

h is inversely proportional to p

p is directly proportional to $\sqrt{t}$

Given that $h = 10$ and $t = 144$ when $p = 6$
find a formula for h in terms of t

h	p	$\sqrt{t}$	t
10	6	12	144
	$\frac{1}{2}\sqrt{t}$	$\sqrt{t}$	

$\times\frac{1}{2}$

$$\begin{cases} hp=60 \\ p=\frac{1}{2}\sqrt{t} \end{cases}$$

$h\times\frac{1}{2}\sqrt{t}=60 \Rightarrow h=\frac{120}{\sqrt{t}}$